
Building
Expressions
3
FENCE

CHAPTER 3 Building Expressions

You may have heard the saying, "Mathematics is a universal language." This is sometimes said because mathematics uses symbols (like letters and numbers) to describe mathematical ideas and relationships in **general** ways to understand specific cases. For example, a statement like, "*I am exactly 3 years older than my cousin,*" describes a **general** relationship that is true for any specific age of your cousin. To represent **general** relationships like this, you will use variables (like x) and build expressions (like $x+3$).

Section 3.1 begins building expressions by using a new tool called "algebra tiles." You will use a variable to help you describe the perimeter and area of shapes built with tiles when one dimension is unknown or can represent various lengths.

Guiding Questions

Think about these questions throughout this chapter:

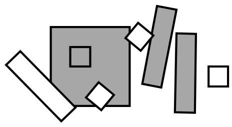
How can I write it?

How can I represent the relationship?

How can I organize my thinking?

Are these representations equivalent?

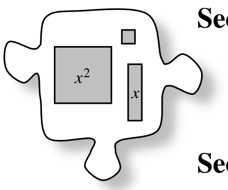
In Section 3.2, you will use variables to represent a single unknown number in different contexts, such as the balance of your bank account or the number of minutes you spend doing homework. You will learn about the 5-D Process, a **strategy** to organize your thinking that can help you solve problems.



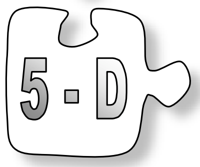
In this chapter, you will learn how to:

- Combine like terms and simplify variable expressions.
- Use a variable to represent any number.
- Substitute a given value for a variable and evaluate an expression.
- Solve situational problems using the 5-D Process.

Chapter Outline

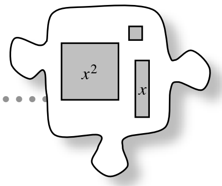


Section 3.1 This section introduces algebra tiles and uses their area and perimeter to develop the skills of building expressions and combining like terms. Variable expressions will be simplified and evaluated for given values.



Section 3.2 This section will introduce the 5-D Process as a problem-solving method. You will learn how to understand a problem by drawing, describing, and defining its elements. You will learn **strategies** that will help you throughout the course and lead to writing and solving equations later in the course.

3.1.1 What is the area?



Area of Rectangular Shapes

Mathematics can be used to describe patterns in the world. Scientists use math to describe various aspects of life, including how cells multiply, how objects move through space, and how chemicals react. Often, when scientists try to describe these patterns, they need to describe something that changes or varies. Scientists call those quantities that change **variables**, and represent them using letters and symbols.

In this course you will spend time learning about variables, what they can represent, and how they can serve different purposes. To start, you will use variables to describe the dimensions and area of different shapes and begin to organize those descriptions into **algebraic expressions**.

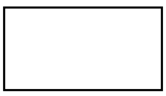
As you work with your teammates, use the following questions to help focus your team's discussion:

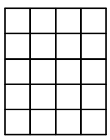
How can we organize groups of things?

What is the area?

Which lengths can vary?

3-1. Find the area of each rectangle below. Show your work. In part (b), each small square in the interior of the rectangle represents one square unit.

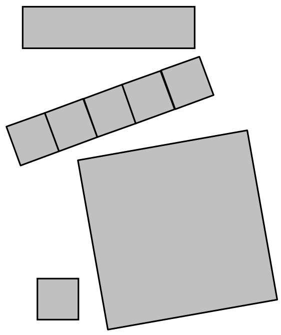
a. 15 cm  25 cm

b. 

c. 0.75 m  1.8 m

d. Explain your method for finding the area of a rectangle.

3-2. AREA OF ALGEBRA TILES

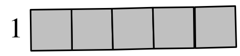


Your teacher will provide your team with a set of algebra tiles. Remove one of each shape from the bag and put it on your desk. Trace around each shape on your paper. Look at the different sides of the shapes.

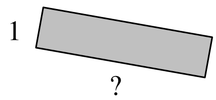
a. With your team, discuss which shapes have the same side lengths and which ones have different side lengths. Be prepared to share your ideas with the class. On your traced drawings, color code lengths that are the same.

b. Each type of tile is named for its area. In this course we will say that the smallest square has a side length of 1 unit, so its area is 1 square unit. We will call this tile "one" or the "unit tile." Can you use the unit tile to find the side lengths of the other rectangles? Why or why not?

c. If the side lengths of a tile can be measured exactly, then the area of the tile can be calculated by multiplying these two lengths together. The area is measured in square units. For example, the tile at right measures 1 unit by 5 units, so it has an area of 5 square units.



The next tile at right has one side length that is exactly one unit long. If we cannot give a numerical value to the other side length, what can we call it?

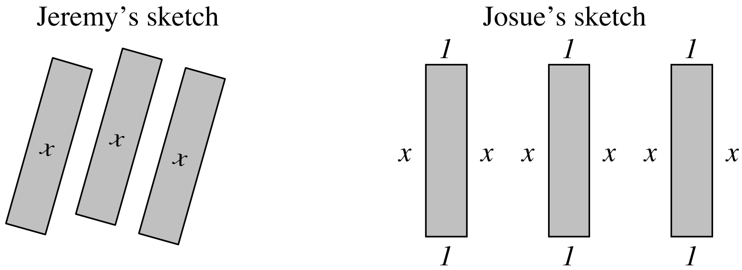


d. If we agree to call the unknown length "*x*," label the side lengths of each of the four algebra tiles you traced. Find each area and use it to name each tile. Be sure to include the name of the type of units it represents.

3-3. Jeremy and Josue have each sketched three x-tiles on their papers.

Jeremy has labeled each tile with an x. *"They are three x-tiles, each with dimensions 1 by x, so the total area is 3x square units,"* he said.

Josue labeled the dimensions, that is, length and width, of each tile. His sketch shows six x-lengths.



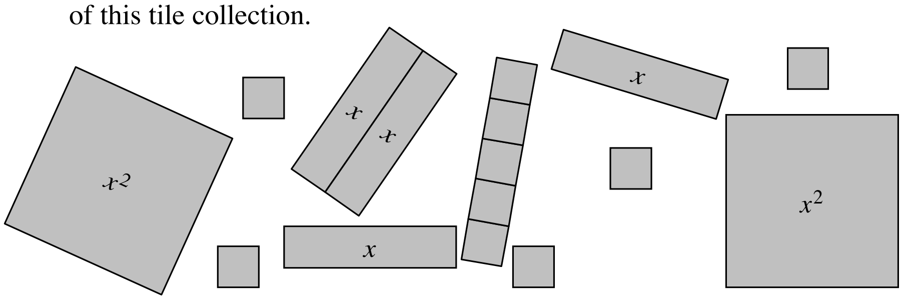
a. Why do the two sketches each show a different number of x labels on the shapes?

b. When tiles are named by their area, they are named in square units. For example, the x by x tile (large square) is a shape that measures x^2 square units of area, so we call it an x^2-tile.

   What do the six x's on Josue's sketch measure? Are they measures of square units?

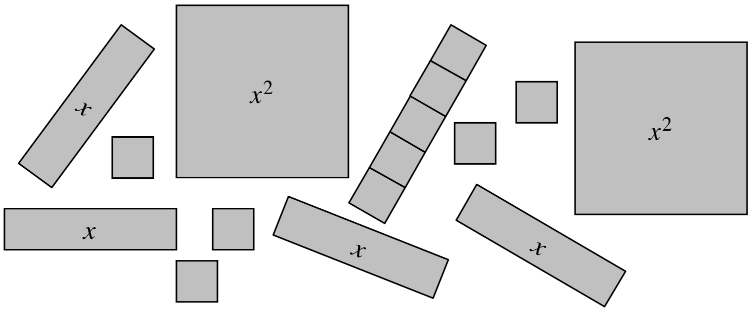
3-4. When a collection of algebra tiles is described with mathematical symbols it is called a **variable expression**. Take out the tiles shown in the picture below and put them on your table.

- Use mathematical symbols (numbers, variables, and operations) to record the area of this collection of tiles.
- Write at least three different variable expressions that represent the area of this tile collection.

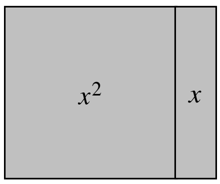


3-5. Take out the tiles pictured in each collection below, put them on your table, and work with your team to find the area as you did in problem 3-4.

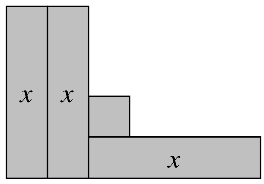
a.



b.



c.



d. Is the area you found in part (a) the same or different from the area of the collection in problem 3-4? **Justify** your answer using words, pictures, or numbers.

3-6. Build each collection described below with algebra tiles and use them to help you to answer the questions.

a. If a person combined a collection of three x^2-tiles, two x-tiles and five unit tiles with one x^2-tile and two x-tiles, how many of each tile would they have?

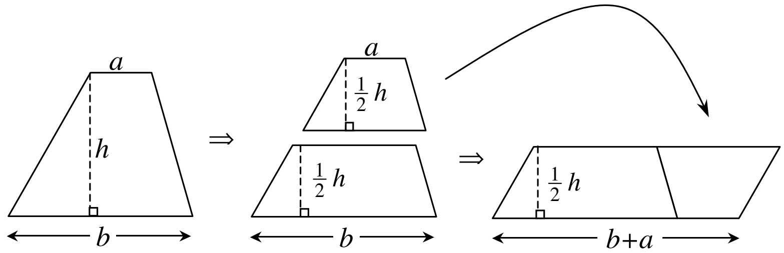
b. If a student started with three x^2-tiles, two x-tiles and five unit tiles and removed two x^2-tiles, two x-tiles and three unit tiles, what would remain?

MATH NOTES

METHODS AND MEANINGS

Area of Trapezoids

There are multiple ways to divide a trapezoid and rearrange the pieces into a parallelogram with the same area. For example, the trapezoid can be divided parallel to its two bases to create two smaller trapezoids that are each half of the height of the original trapezoid. Those two pieces can be rearranged into a parallelogram, as shown below.

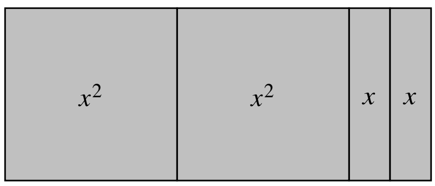


Therefore, to find the **area of a trapezoid**, find the product of half of the height (*h*) and the sum of the two bases (*a* and *b*).

$$A = \frac{1}{2}h(a + b)$$

Review & Preview

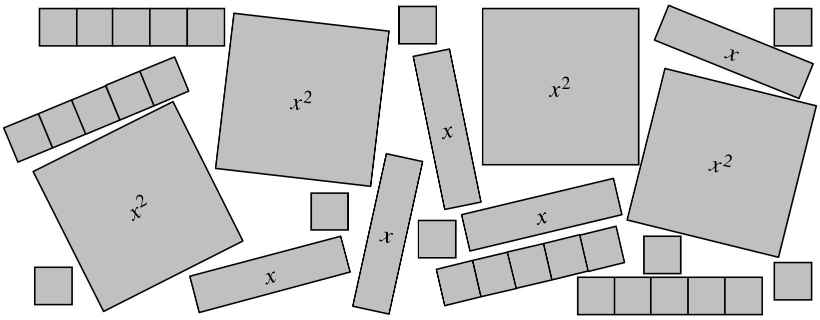
3-7. Sketch the shape made with algebra tiles at right on your paper. Then answer parts (a) and (b) below.



a. Find the area of the shape.

b. If the algebra tiles were rearranged into a different shape, how would the area change?

3-8. Your team forgot to clean up their algebra tiles and now they are all over your desk.

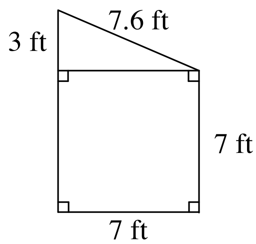


Sort the tiles so that they are in groups that are all the same and write a sentence explaining how many of each type of tile you have.

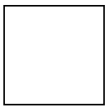
3-9. Suppose that a triangle is created on a graph by connecting points *A*, *B*, and *C* below. Marika, Kimiku and Marcus want to move triangle *ABC* five units to the left and four units down. Write an integer expression to record how the shape moves and find the new coordinates for each vertex.

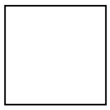
a. $A(-1, 3)$ b. $B(5, -1)$ c. $C(0, 1)$

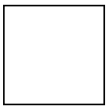
3-10. Find the area and perimeter of the shape at right.

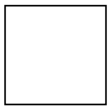


3-11. Find the area of each square with the given side lengths.

a. 

b. 

c. 

d. 

e. Describe the method you used to find the area of the squares.